MOMENTI D'INERZIA

$I=\frac{bH^3}{36}$

$I=\frac{H^4}{12}$

$I=\frac{BH^3}{12}$

$I=\frac{\pi D^4}{64}$

1 ft = 0,3048 m
1 stoke = 1 cm²/s (ν)
1 poise = 0,1 Pa·s (μ)

- $V=\frac{M}{\rho}$ • $\dot m=\rho\dot V=\rho A|\vec v|$ • $\dot V=A|\vec v|$
- $p=\rho RT$ • $M_a=\frac{V_{fluido}}{c_{suono}}$ • $SG=\frac{\rho}{\rho_{H_2O}}$
- $\rho_{H_2O}(4°C)=1000\ kg/m^3$ – $R=8{,}314\ \frac{\cancel{k}J}{\cancel{k}mol\,K}$
- $c_{suono}=346$ m/s in aria
- $c_{luce}=2{,}99\cdot10^8$ m/s in vuoto

COMPRESS./DILATAZ.

- $\frac{d\rho}{\rho}=-\frac{dv}{v}=\alpha_T dp-\beta\, dT$ $\alpha=\frac{1}{k}$

STATICA DEI FLUIDI

- $\vec\nabla p=\rho\vec g=(0,0,-\rho g)$ GRAVITÀ ZERO: $\vec\nabla p=\vec 0$

STATICA NON INERZIALE

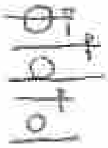
$\vec\nabla p=\rho(\vec g-\vec a)=(-\rho a_x,-\rho a_y,-\rho a_z-\rho g)$, PENDENZA ISOBARE $=-\frac{a_x}{g}$ " FORZE $=\frac{g}{a_x}$ } ⊥

- ACCELERAZ. LUNGO 1 DIREZIONE: $z=-\frac{a_x}{g}x+C$
- ACCELERAZ. COSTANTE: $z=-\frac{a_x}{g+a_z}(x-x_0)+z_0$ [illegible]
- ROTAZIONE A ω COSTANTE: $z=\frac{\omega^2r^2}{2g}+h_c(\omega)$, $z=\frac{\omega^2r^2}{2g}+\underbrace{h_0-\frac{\omega^2R^2}{4g}}_{h_c(\omega)}$ [illegible] $f(\omega)$)

LEGGE DI ARCHIMEDE

- $\vec F_{arch}=\rho_{fluido}\vec g\cdot V_s$ ↗ VOLUME OCCUPATO DALL'OGGETTO (IMMERSO NEL FLUIDO)

$|F_{arch}|>|W|$
$|F_{arch}|<|W|$
$|F_{arch}|=|W|$



LEGGE YOUNG-LAPLACE

- $p_{int}-p_{est}=\sigma_s\left(\frac{1}{R'}+\frac{1}{R''}\right)$ R', R'' RAGGI DI CURVATURA (SFERA: R' = R'' = R)

CAPILLARITÀ

$\sigma_s 2\pi R\cos\varphi=\rho g\pi r^2h\rightarrow h=\frac{2\sigma_s\cos\varphi}{\rho g r}=\frac{4\sigma_s\cos\varphi}{\rho g d}$ $0<\varphi<90°$ $90°<\varphi<180°$

LEGGE DI PASCAL

$\frac{F_1}{A_1}=\frac{F_2}{A_2}$ (MARTINETTO IDRAULICO) $\Rightarrow p_1=p_2\Rightarrow F_1=\frac{A_1}{A_2}F_2$ $\frac{A_1}{A_2}$ VANTAGGIO MECCANICO IDEALE

DINAMICA DEI FLUIDI

- CAMPO ACCELERAZ.: $\vec a=\frac{D\vec V}{Dt}=\frac{\partial\vec V}{\partial t}+\vec V\cdot\vec\nabla\vec V=\frac{\partial}{\partial t}(u,v,w)+u\frac{\partial\vec V}{\partial x}+v\frac{\partial\vec V}{\partial y}+w\frac{\partial\vec V}{\partial z}$
- DERIVATA MATERIALE: $\frac{DB}{Dt}=\frac{\partial}{\partial t}(B)+u\frac{\partial B}{\partial x}+v\frac{\partial B}{\partial y}+w\frac{\partial B}{\partial z}$
- LINEE DI CORRENTE: $\frac{dr}{V}=\frac{dx}{u}=\frac{dy}{v}=\frac{dz}{w}$
- TRAIETTORIA: $dt=\frac{dx}{u}=\frac{dy}{v}=\frac{dz}{w}$

} IN REGIME STAZ. COINCIDONO E COINCIDONO CON LE LINEE DEL FUMO

- VELOCITÀ ANGOLARE: $\omega=\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$
- $\varepsilon_{xx}=\frac{\partial u}{\partial x}$, $\varepsilon_{yy}=\frac{\partial v}{\partial y}$, $\varepsilon_{zz}=\frac{\partial w}{\partial z}$ (DEF. LINEARE) $\frac{dV_{sist}}{dt}=\overset{\varepsilon_{xx}}{\overset{\|}{\frac{\partial u}{\partial x}}}+\overset{\varepsilon_{yy}}{\overset{\|}{\frac{\partial v}{\partial y}}}+\overset{\varepsilon_{zz}}{\overset{\|}{\frac{\partial w}{\partial z}}}=\vec\nabla\cdot\vec V=div(\vec V_{el})\Rightarrow$ SE $div(\vec V)=0$, INCOMPRIM.
- $\varepsilon_{xy}=\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)=\varepsilon_{yx}$, $\varepsilon_{xz}=\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)=\varepsilon_{zx}$, $\varepsilon_{yz}=\frac{1}{2}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)=\varepsilon_{zy}$ (DEF. ANGOLARE)
- $\vec\nabla\vec V=\begin{pmatrix}\frac{\partial u}{\partial x}&\frac{\partial v}{\partial x}&\frac{\partial w}{\partial x}\\\frac{\partial u}{\partial y}&\frac{\partial v}{\partial y}&\frac{\partial w}{\partial y}\\\frac{\partial u}{\partial z}&\frac{\partial v}{\partial z}&\frac{\partial w}{\partial z}\end{pmatrix}$
- $\vec{\vec\varepsilon}=\frac{1}{2}\left(\vec\nabla\vec V+(\vec\nabla\vec V)^T\right)$ – SIMMETRICO · TENSORE DI VEL. DEF.
- $\vec{\vec\Omega}=\frac{1}{2}\left(\vec\nabla\vec V-(\vec\nabla\vec V)^T\right)$ – ANTISIMMETRICO · TENSORE DI VEL. ANG.

} $\Rightarrow\vec V_P=\vec V_0+\vec X\cdot\vec{\vec\varepsilon}+\vec X\cdot\vec{\vec\Omega}$ $\vec X=(P-O)$

- VORTICITÀ $\vec\zeta=rot\,\vec V_{el}=det\begin{pmatrix}e_1&e_2&e_3\\\frac{\partial}{\partial x}&\frac{\partial}{\partial y}&\frac{\partial}{\partial z}\\u&v&w\end{pmatrix}$, $\vec\omega=\frac{1}{2}\vec\zeta$

$\vec\zeta\begin{cases}<0 & ↻\\>0 & ↺\end{cases}$

$\vec\zeta=\underline{e_1}\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)+\underline{e_2}\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)+\underline{e_3}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$

$\vec\zeta=\underline{e_r}\left(\frac{1}{r}\frac{\partial u_z}{\partial\theta}-\frac{\partial u_\theta}{\partial z}\right)+\underline{e_\theta}\left(\frac{\partial u_r}{\partial z}-\frac{\partial u_z}{\partial r}\right)+\underline{e_z}\left(\frac{\partial(r u_\theta)}{\partial r}-\frac{\partial u_r}{\partial\theta}\right)$

STATICA FLUIDI (II)

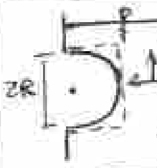
FORZA PRESS.

$F_R=\rho g h_c\cdot A$

MOMENTI

$y_P=y_C+\frac{I_{xx,C}}{y_C\cdot A}$ IN y_P = RETTA D'AZIONE

STATICA FLUIDI (III)



$F_z=\pm F_{ARCHIM.}$

$F_x=p_C\cdot A=p_C\cdot2Rb$

$\alpha=\tan^{-1}\frac{F_z}{F_x}$ $M=|F|R\sin\alpha$

MANOMETRO DIFFERENZ.

- $p_1-p_2=gh(\rho_2-\rho_1)$ $(\rho_2>\rho_1)$

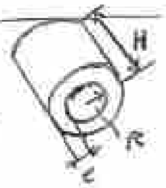
MOTO DI COUETTE

$\tau=\mu\frac{du}{dy}$ con $u=u_{max}\frac{y}{h}$

$\tau[Pa]$

$\tau=\mu\frac{u}{h}$

VISCOSIMETRO



$M=F\cdot R=\tau\cdot R\cdot2\pi RH$

$M=2\pi R^2H\cdot\mu\frac{V}{\ell}$

$\Rightarrow\mu=\frac{M\ell}{2\pi R^2HV}=\frac{M\ell}{4\pi^2R^3nH}$

$(V=2\pi nR)$

CIRCOLAZIONE

$\Gamma = \oint_C \vec{V} d\vec{l} = \iint_A rot(\vec{V}) \cdot \vec{n}\, dA$ TEOREMA DI STOKES

INT. LINEA $\int_\gamma \vec{F} \cdot \vec{t} = \int_a^b F(\gamma(t)) \cdot \gamma'(t) dt$

$\Gamma \rightleftarrows \vec{\zeta}$: se $\Gamma = 0$ allora $\vec{\zeta} = \vec{0}$ e viceversa

R.T.T. (PASSA DA TRATTAZ. LAGRANGIANA A EULERIANA, DA INTEGRALE A DIFFERENZIALE, DALLE PARTICELLE AL V.C.)

B_{sys} = PROPRIETÀ SISTEMA $\vec{V}_{vc} = \vec{V}_{sc}$

• $\frac{DB_{sys}}{Dt} = \frac{d}{dt}B_{sys} + \dot{B}_{out} - \dot{B}_{in}$ • $\frac{DB_{syst}}{Dt} = \frac{dB_{sys}}{dt} + \int_{S.C.} \rho \vec{V} \cdot \vec{n}\, dA$ • $\frac{DB_{sys}}{Dt} = \frac{d}{dt}\int_{V.C.} \rho b\, dV + \int_{S.C.} b \rho \vec{V} \cdot \vec{n}\, dA$

V.C. $\vec{V}_r = \vec{V} - \vec{V}_c = \begin{cases} \vec{V} - \vec{V}_c & \text{IMP} \\ \vec{V} - \vec{V}_{sc} & \text{DEF} \\ 0 & \text{FISSO} \end{cases}$

se V.C. FISSO

$\frac{DB_{sys}}{Dt} = \int_{V.C.} \frac{d}{dt}(\rho b) dV + \int_{S.C.} \rho \vec{V} \cdot \vec{n}\, dA$

• se V.C. si muove, $\vec{V} = \vec{V}_r = \vec{V} - \vec{V}_{V.C.}$

• se $B_{sys} = m$, $\frac{DB_{sys}}{Dt} = 0 \Rightarrow \frac{d\rho}{dt} + \vec{\nabla}(\rho \vec{V}) = 0$

$\frac{d\rho}{dt} + \rho \vec{\nabla} \vec{V} + \vec{V} \vec{\nabla} \rho = 0$

• $\frac{1}{\rho}\frac{D\rho}{Dt} + \vec{\nabla} \vec{V} = 0$ eq^{ne} conservaz. massa

• IN UN SISTEMA FLUIDO LA MASSA SI CONSERVA, NON IN UN V.C.

• $\int_{S.C.} \rho \vec{V} \cdot \vec{n}\, dA + \frac{d}{dt}\int_{V.C.} \rho\, dV = 0$ (CONS. MASSA) se moto staz. $\sum_{in} \dot{m} = \sum_{out} \dot{m}$

$\dot{W}_{turb} = \dot{m} g h_{turb}$; $\dot{W}_{pompa} = \dot{m} g h_{pompa}$ e incompr. $\sum_{in} \dot{V} = \sum_{out} \dot{V}$

$\dot{W}$ = POTENZA = $\dot{m} g h = \dot{m} g e$, $e = \frac{V^2}{2} + \frac{P}{\rho} + \rho z$ [J/kg] $\boxed{E = \rho e}$

$\eta = \frac{\text{EN. USCITA}}{\text{EN. INGRESSO}} = \frac{\text{POT. USCITA}}{\text{POT. INGRESSO}}$ (< 1) (η MECCANICO)

$\eta_{turb} = \frac{\text{EN. MECCANICA}}{\text{EN. FLUIDO}} = \frac{\text{EN. MEC.}}{\text{LAVORO SU ALBERO}} \cdot \frac{\text{LAVORO}}{\text{EN. FLUIDO}} = \eta_{elettrico} \cdot \eta_{fluido}$ (η MOTORE)

$\eta_{pompa} = \frac{\text{EN. FLUIDO}}{\text{EN. ELETTRICA}} = \frac{\text{EN. FLUIDO}}{\text{LAVORO ALBERO}} \cdot \frac{\text{LAVORO ALBERO}}{\text{EN. MECCANICA}} = \eta_{fluido} \cdot \eta_{elettrico}$

• BERNOULLI (FLUIDO STAZIONARIO, INCOMPR., ESENTE ATTRITI)

• $\frac{P}{\rho} + \frac{V^2}{2} + \rho z = cost$

• $\frac{P}{\rho g} + \frac{V^2}{2g} + z = H$ = CARICO TOTALE

BERNOULLI VALE SU LINEE DI CORRENTE

$h_{pompa} = \frac{\Delta P_{pompa}}{\rho g}$

CONSERVAZ. DELL'ENERGIA

$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + h_{pompa} + z_1 = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + h_{turbina} + \overset{\text{PERDITE DI CARICO}}{h_L} + z_2$

• LINEA CARICHI PIEZOM. E LINEA EN. MECC. sono // e

SFASATE DI $\frac{V^2}{2g}$
- INIZIANO A $P_1 = 0$ (LIV. PELO LIBERO)
- FINISCONO [illegible] CURVATE

PIEZOMETRO (PITOT)

• $v = \sqrt{\frac{2(P_2 - P_1)}{\rho}}$

• $v = \sqrt{2g(\underbrace{h_3 - h_1}_{\Delta})}$

CONSERVAZ. Q. DI MOTO

• $\sum \vec{F} = \frac{d}{dt}\int_{V.C.} \rho \vec{V} dV + \int_{S.C.} \rho \vec{V} (\vec{V} \cdot \vec{n}) dA$, se V.C. FISSO • $\sum \vec{F} = \int_{V.C.} \frac{d}{dt}(\rho \vec{V}) dV + \int_{S.C.} \rho \vec{V} (\vec{V} \cdot \vec{n}) dA$

• $\sum \vec{F} = \frac{d}{dt}\int_{V.C.} \rho \vec{V} dV + \sum_{out} \dot{m} \beta \vec{V}_m - \sum_{in} \dot{m} \beta \vec{V}_m$

- $\beta = \frac{\vec{V} \int \rho \vec{V} \cdot \vec{n}\, dA}{\dot{m} \vec{V}}$
- $\beta = \frac{4}{3}$ LAMINARE
- $\beta = 1$ MOTO UNIFORME
- $\beta = 1,05$ REGIME TURBOLENTO

CONSERVAZ. MOM. ANGOLARE $\vec{\omega} = \frac{2\pi \dot{n}}{60}$, $\dot{W} = \frac{2\pi \dot{n}}{60}|\vec{M}|$

• $\sum \vec{M} = \frac{d\vec{H}}{dt} = \frac{d}{dt}\int_{V.C.} \rho(\vec{r} \times \vec{V}) dV + \int_{S.C.} \rho(\vec{r} \times \vec{V})(\vec{V} \cdot \vec{n}) dA$

$= \int_{V.C.} \frac{d}{dt}[\rho(\vec{r} \times \vec{V})] dV +$ " (V.C. FISSO)

$\sum \vec{M} = \sum_{out} \vec{r} \times \dot{m} \vec{V} - \sum_{in} \vec{r} \times \dot{m} \vec{V}$ (STAZIONARIO) → $\sum M = \sum_{out} r \dot{m} |\vec{V}| - \sum_{in} r \dot{m} |\vec{V}|$ (SCALARE)

10^{12}	TERA
10^{9}	GIGA
10^{6}	MEGA
10^{3}	KILO
10^{2}	ETTO
10^{1}	DECA
10^{-1}	DECI
10^{-2}	CENTI
10^{-3}	MILLI
10^{-6}	MICRO
10^{-9}	NANO
10^{-12}	PICO

(ANTIPANICO)

VOL CONO $= \frac{1}{3}\pi R^2 h$ SUP CONO $= \pi R A$

VOL CIL $= \pi R^2 \ell$

• PRESTARE ATTENZIONE A
- UNITÀ DI MISURA
- GEOMETRIA DEL SISTEMA (SIMMETRIE, INCLINAZIONI, PARTI...)
- DATI DI PRESSIONE TRASCURABILI
- MOMENTI DI FORZE E ANGOLARI
- PROB. ISTANTANEA
- CONVENZIONE FORZE COME ⊕, POI VERIFICARE SEGNI